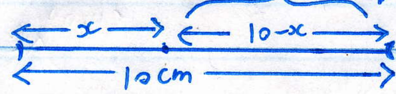
② A piece of wire 10cm long is cut into pieces. One pieces is bent into a circle and the other is bent into equilateral triangle. How should the wire be cut so that the total area that is enclosed is minimal? maximal?

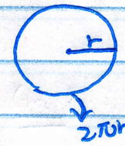
Ans

Forms the circle.




Forms the equilateral triangle

The equilateral triangle. therefore has sides end of triangle $\frac{2}{3}$ cm

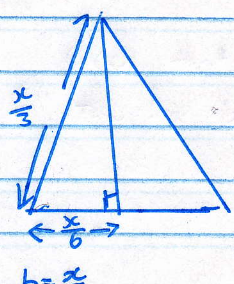



Circumference of the circle is $(10-x) = 2\pi r$, r = radius of the circle.

Total enclosed area = Area enclosed by the equilateral $\Delta$

+

Area enclosed by the circle.




$\frac{1}{2}bh$

$h^2 = (\frac{x}{3})^2 - (\frac{x}{6})^2$

Recall that the area of the triangle = $\frac{1}{2} \cdot \frac{x}{3} \cdot \frac{x\sqrt{3}}{6}$

Total enclosed area = Area enclosed by the eqilateral $\Delta = (\frac{1}{2})(\frac{x}{3})(\frac{x\sqrt{3}}{6})$

+

Area enclosed by the circle $= \pi r^2 = \frac{\pi(10-x)^2}{4\pi^2} = \frac{(10-x)^2}{4\pi}$

Hence total enclosed area $= \frac{1}{12}(\frac{x^2}{\sqrt{3}}) + \frac{(10-x)^2}{4\pi} = A(x)$

$A'(x) = \frac{1}{\cancel{12}_{6}\sqrt{3}} \times 2x + \frac{\cancel{2}(10-x)(-1)}{\cancel{4}_{2}\pi} = \frac{2}{6\sqrt{3}} - \frac{(10-x)}{2\pi}$